2.44. (a)

$$
\begin{aligned}
X(e^{j\omega})|_{\omega=0} &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}|_{\omega=0} \\
&= \sum_{n=-\infty}^{\infty} x[n] \\
&= 6
\end{aligned}
$$

(b)

$$
\begin{aligned}
X(e^{j\omega})|_{\omega=\pi} &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\pi n} \\
&= \sum_{n=-\infty}^{\infty} x[n](-1)^n \\
&= 2
\end{aligned}
$$

(c) Because $x[n]$ is symmetric about $n = 2$ this signal has linear phase.

$$X(e^{j\omega}) = A(\omega)e^{-j2\omega}$$

$A(\omega)$ is a zero phase (real) function of ω. Hence,

$$\angle X(e^{j\omega}) = -2\omega, \quad -\pi \leq \omega \leq \pi$$

(d)

$$\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega n}d\omega = 2\pi x[n]$$

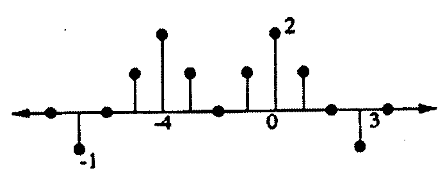
for $n = 0$:

$$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 2\pi x[0] = 4\pi$$

(e) Let $y[n]$ be the unknown sequence. Then

$$
\begin{aligned}
Y(e^{j\omega}) &= X(e^{-j\omega}) \\
&= \sum_n x[n]e^{j\omega n} \\
&= \sum_n x[-n]e^{-j\omega n} \\
&= \sum_n y[n]e^{-j\omega n}
\end{aligned}
$$

Hence $y[n] = x[-n]$.



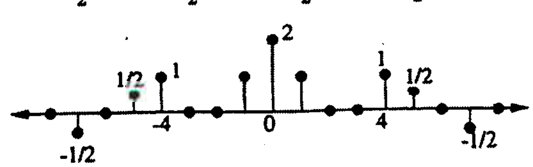
(f) We have determined that:

$$X(e^{j\omega}) = A(\omega)e^{-j2\omega}$$

$$
\begin{aligned}
X_R(e^{j\omega}) &= \mathcal{R}e\{X(e^{j\omega})\} \\
&= A(\omega)\cos(2\omega) \\
&= \frac{1}{2}A(\omega)\left(e^{j2\omega} + e^{-j2\omega}\right)
\end{aligned}
$$

Taking the inverse transform, we have

$$\frac{1}{2}a[n+2] + \frac{1}{2}a[n-2] = \frac{1}{2}x[n+4] + \frac{1}{2}x[n]$$



2.45. Let $x[n] = \delta[n]$, then

$$X(e^{j\omega}) = 1$$

The output of the ideal lowpass filter:

$$W(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = H(e^{j\omega})$$

The multiplier:

$$(-1)^n w[n] = e^{-j\pi n} w[n]$$

causes a shift in the frequency domain:

$$W(e^{j(\omega-\pi)}) = H(e^{j(\omega-\pi)})$$

The overall output:

$$y[n] = e^{-j\pi n} w[n] + w[n]$$
$$Y(e^{j\omega}) = H(e^{j(\omega-\pi)}) + H(e^{j\omega})$$

Noting that:

$$H(e^{j(\omega-\pi)}) = \begin{cases} 1, & \frac{\pi}{2} \le |\omega| \le \pi \\ 0, & |\omega| < \frac{\pi}{2} \end{cases}$$

$Y(e^{j\omega}) = 1$, thus $y[n] = \delta[n]$.

2.51

a) $y_1[n] = x_1[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] x_1[n-k] =$

$= \sum_{k=-\infty}^{\infty} a^k u[k] e^{j(\frac{\pi}{2})(n-k)} = e^{j\frac{\pi}{2}n} \sum_{k=0}^{\infty} (a e^{-j\frac{\pi}{2}})^k = \frac{e^{j\frac{\pi}{2}n}}{1 - a e^{-j\frac{\pi}{2}}} \Rightarrow$

$y_1[n] = \frac{e^{j\frac{\pi}{2}n}}{1 + ja} \qquad |a|<1$

b) $y_2[n] = x_2[n] * h[n] = \cos(\frac{\pi}{2}n) * h[n] = \left(\frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}\right) * h[n]$

$= \frac{1}{2}\underbrace{\left(e^{j\frac{\pi}{2}n} * h[n]\right)}_{\frac{e^{j\frac{\pi}{2}n}}{1+ja}} + \frac{1}{2}\underbrace{\left(e^{-j\frac{\pi}{2}n} * h[n]\right)}_{\frac{e^{-j\frac{\pi}{2}n}}{1-ja}} \Rightarrow$

$y_2[n] = \frac{1}{2}\left(\frac{e^{j\frac{\pi}{2}n}}{1+ja} + \frac{e^{-j\frac{\pi}{2}n}}{1-ja}\right)$

c) $y_3[n] = x_3[n] * h[n] = \sum_{k=-\infty}^{\infty} a^k u[k] e^{j\frac{\pi}{2}(n-k)} u[n-k] =$

$= \sum_{k=0}^{n} a^k e^{j\frac{\pi}{2}(n-k)} = e^{j\frac{\pi}{2}n} \sum_{k=0}^{n} (a e^{-j\frac{\pi}{2}})^k \Rightarrow$

$y_3[n] = e^{j\frac{\pi}{2}n} \frac{1 - (a e^{-j\frac{\pi}{2}})^{n+1}}{1 + ja}$

d) since $\lim_{n\to\infty} (a e^{-j\frac{\pi}{2}})^{n+1} \to 0$, $\lim_{n\to\infty} y_3[n] = \lim_{n\to\infty} y_1[n]$

So for $n \to \infty$, $y_3[n] = y_1[n]$

2.53. First $x[n]$ goes through a lowpass filter with cutoff frequency 0.5π. Since the cosine has a frequency of 0.6π, it will be filtered out. The delayed impulse will be filtered to a delayed sinc and the constant will remain unchanged. We thus get:

$$w[n] = 3\frac{sin(0.5\pi(n-5))}{\pi(n-5)} + 2.$$

$y[n]$ is then given by:

$$y[n] = 3\frac{sin(0.5\pi(n-5))}{\pi(n-5)} - 3\frac{sin(0.5\pi(n-6))}{\pi(n-6)}.$$

2.54.

$$\begin{aligned} x[n] &= \cos(\frac{15\pi n}{4} - \frac{\pi}{3}) \\ &= \cos(-\frac{\pi n}{4} - \frac{\pi}{3}) \\ &= \cos(\frac{\pi n}{4} + \frac{\pi}{3}) \\ &= \frac{e^{j\frac{\pi}{3}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{\pi}{3}} e^{-j\frac{\pi n}{4}}}{2}. \end{aligned}$$

Using the fact that complex exponentials are eigenfunctions of LTI systems, we get:

$$\begin{aligned} y[n] &= e^{-j\frac{3\pi}{8}} \frac{e^{j\frac{\pi}{3}} e^{j\frac{\pi n}{4}}}{2} + e^{-j\frac{\pi}{8}} \frac{e^{-j\frac{\pi}{3}} e^{-j\frac{\pi n}{4}}}{2} \\ &= \frac{e^{-j\frac{\pi}{24}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{11\pi}{24}} e^{-j\frac{\pi n}{4}}}{2} \\ &= e^{-j\frac{\pi}{4}} (\frac{e^{j\frac{5\pi}{24}} e^{j\frac{\pi n}{4}}}{2} + \frac{e^{-j\frac{5\pi}{24}} e^{-j\frac{\pi n}{4}}}{2}) \\ &= e^{-j\frac{\pi}{4}} \cos(\frac{\pi n}{4} + \frac{5\pi}{24}). \end{aligned}$$

2.66. (a)

$$\begin{aligned} E(e^{j\omega}) &= H_1(e^{j\omega})X(e^{j\omega}) \\ F(e^{j\omega}) &= E(e^{-j\omega}) \\ &= H_1(e^{-j\omega})X(e^{-j\omega}) \\ G(e^{j\omega}) &= H_1(e^{j\omega})F(e^{j\omega}) \\ &= H_1(e^{j\omega})H_1(e^{-j\omega})X(e^{-j\omega}) \\ Y(e^{j\omega}) &= G(e^{-j\omega}) \\ &= H_1(e^{-j\omega})H_1(e^{j\omega})X(e^{j\omega}). \end{aligned}$$

(b) Since:

$$Y(e^{j\omega}) = H_1(e^{-j\omega})H_1(e^{j\omega})X(e^{j\omega}),$$

We get:

$$H(e^{j\omega}) = H_1(e^{-j\omega})H_1(e^{j\omega}).$$

(c) Taking the inverse transform of $H(e^{j\omega})$, we get:

$$h[n] = h_1[-n] * h_1[n].$$

2.68. If $x_1[n] = x_2[n]$, $w_1[n]$ and $w_2[n]$ will not be necessarily equal.

$$\begin{aligned} w_1[n] &= x_1[-n-2] \\ w_2[n] &= x_2[-n+2] \\ &\neq x_2[-n-2] \end{aligned}$$

A simple counterexample is $x_1[n] = x_2[n] = \delta[n]$. Then:

$$\begin{aligned} w_1[n] &= \delta[n+2] \\ w_2[n] &= \delta[n-2]. \end{aligned}$$