3.23. (a)

$$\begin{aligned} H(z) &= \frac{1-\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \\ &= -4+\frac{5+\frac{7}{2}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \\ &= -4-\frac{2}{1-\frac{1}{2}z^{-1}}+\frac{7}{1-\frac{1}{4}z^{-1}} \\ h[n] &= -4\delta[n]-2\left(\frac{1}{2}\right)^n u[n]+7\left(\frac{1}{4}\right)^n u[n] \end{aligned}$$

(b)

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]-\frac{1}{2}x[n-2]$$

3.24. The plots of the sequences are shown below.

(a) Let

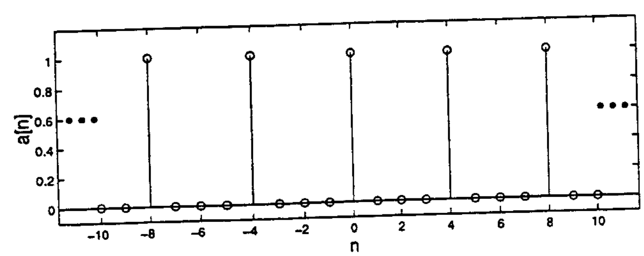
$$a[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k],$$

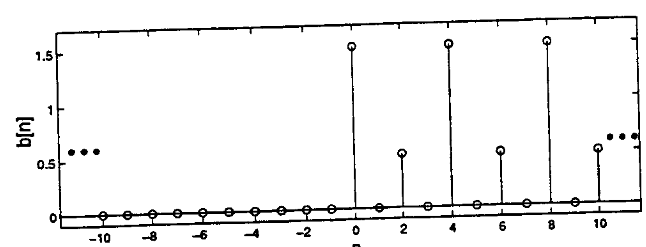
Then

$$A(z)=\sum_{k=-\infty}^{\infty}z^{-4n}$$

(b)

$$\begin{aligned} b[n] &= \frac{1}{2}\left[e^{j\pi n}+\cos\left(\frac{\pi}{2}n\right)+\sin\left(\frac{\pi}{2}+2\pi n\right)\right]u[n] \\ &= \frac{1}{2}\left[(-1)^n+\cos\left(\frac{\pi}{2}n\right)+1\right]u[n] \\ &= \begin{cases} \frac{3}{2}, & n=4k,\ k\geq 0 \\ \frac{1}{2}, & n=4k+2,\ k\geq 0 \\ 0, & \text{otherwise} \end{cases} \\ B(z) &= \sum_{n=0}^{\infty}\frac{3}{2}z^{-4n}+\sum_{n=0}^{\infty}\frac{1}{2}z^{-(4n+2)} \\ &= \frac{3/2+1/2z^{-2}}{1-z^{-4}}, \qquad |z|>1 \end{aligned}$$

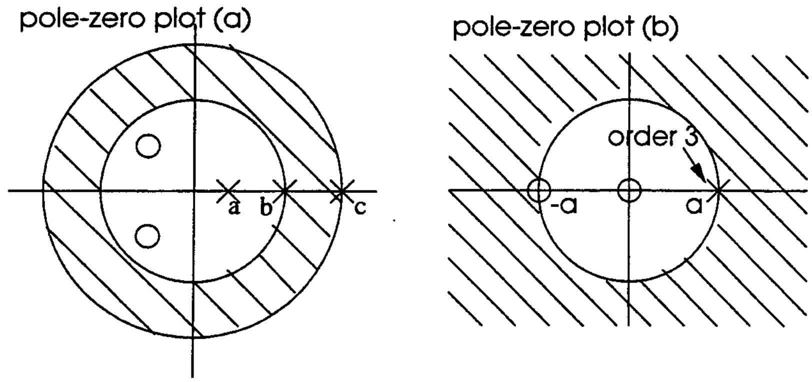




3.31. (a)

$$x[n] = a^n u[n] + b^n u[n] + c^n u[-n-1] \qquad |a| < |b| < |c|$$
$$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}} - \frac{1}{1-cz^{-1}} \qquad |b| < |z| < |c|$$
$$X(z) = \frac{1-2cz^{-1} + (bc+ac-ab)z^{-2}}{(1-az^{-1})(1-bz^{-1})(1-cz^{-1})} \qquad |b| < |z| < |c|$$

Poles: a, b, c,
Zeros: z_1, z_2, ∞ where z_1 and z_2 are roots of numerator quadratic.



(b)

$$x[n] = n^2 a^n u[n]$$
$$x_1[n] = a^n u[n] \Leftrightarrow X_1(z) = \frac{1}{1-az^{-1}} \qquad |z| > a$$
$$x_2[n] = nx_1[n] = na^n u[n] \Leftrightarrow X_2(z) = -z\frac{d}{dz}X_1(z) = \frac{az^{-1}}{(1-az^{-1})^2} \qquad |z| > a$$
$$x[n] = nx_2[n] = n^2 a^n u[n] \Leftrightarrow -z\frac{d}{dz}X_2(z) = -z\frac{d}{dz}\left(\frac{az^{-1}}{(1-az^{-1})^2}\right) \qquad |z| > a$$

(c)

$$x[n] = e^{n^4}\left(\cos\frac{\pi}{12}n\right)u[n] - e^{n^4}\left(\cos\frac{\pi}{12}n\right)u[n-1]$$
$$= e^{n^4}\left(\cos\frac{\pi}{12}n\right)(u[n]-u[n-1]) = \delta[n]$$

Therefore, $X(z) = 1$ for all $|z|$.

3.34.

$$H(z) = \frac{3-7z^{-1}+5z^{-2}}{1-\frac{5}{2}z^{-1}+z^{-2}} = 5 + \frac{1}{1-2z^{-1}} - \frac{3}{1-\frac{1}{2}z^{-1}}$$

$$h[n] \text{ stable } \Rightarrow h[n] = 5\delta[n] - 2^n u[-n-1] - 3\left(\frac{1}{2}\right)^n u[n]$$

(a)

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{n} h[k]$$
$$= \begin{cases} -\sum_{k=-\infty}^{n} 2^k = -2^{n+1} & n < 0 \\ -\sum_{k-\infty}^{-1} 2^k + 5 - \sum_{k=0}^{n} 3\left(\frac{1}{2}\right)^k = 4 - 3\frac{1-(\frac{1}{2})^{n+1}}{1-\frac{1}{2}} = -2 + 3\left(\frac{1}{2}\right)^n & n \geq 0 \end{cases}$$
$$= -2u[n] + 3\left(\frac{1}{2}\right)^n u[n] - 2^{n+1}u[-n-1]$$

(b)

$$Y(z) = \frac{1}{1-z^{-1}}H(z) = -2\frac{1}{1-z^{-1}} + 2\frac{1}{1-2z^{-1}} + 3\frac{1}{1-\frac{1}{2}z^{-1}}, \qquad \frac{1}{2} < |z| < 2$$
$$y[n] = -2u[n] - 2(2)^n u[-n-1] + 3\left(\frac{1}{2}\right)^n u[n]$$