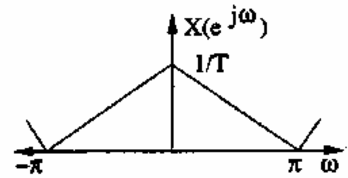
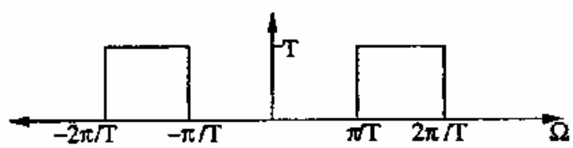
4.22. (a)

$$\omega = \Omega T, \qquad T = \frac{2\pi}{\Omega_0}$$



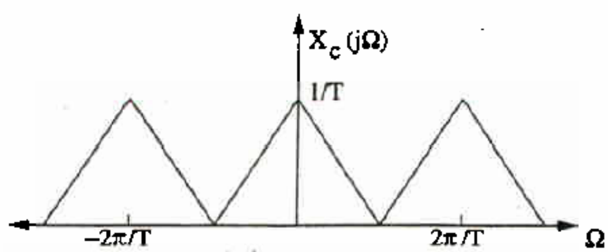
(b) To recover simply filter out the undesired parts of $X(e^{j\omega})$.

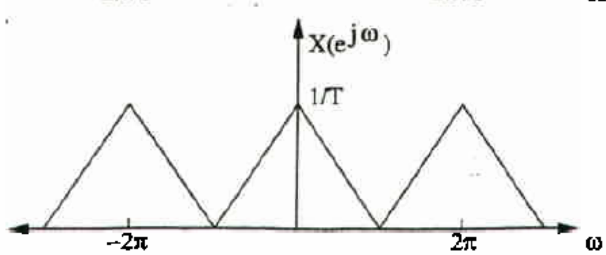


(c)

$$T \leq \frac{2\pi}{\Omega_0}$$

4.25. (a) $x_s(t) = x_c(t)s(t) \Rightarrow X_s(j\Omega) * s(j\Omega)$





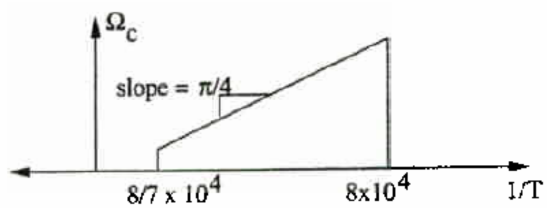
(b) Since $H_d(e^{j\omega})$ is an ideal lowpass filter with $\omega_c = \frac{\pi}{4}$, we don't care about any signal aliasing that occurs in the region $\frac{\pi}{4} \le \omega \le \pi$. We require:

$$\begin{aligned} \frac{2\pi}{T} - 2\pi \cdot 10000 &\ge \frac{\pi}{4T} \\ \frac{1}{T} &\ge \frac{8}{7} \cdot 10000 \\ T &\le \frac{7}{8} \times 10^{-4}\text{sec} \end{aligned}$$

Also, once all of the signal lies in the range $|\omega| \le \frac{\pi}{4}$, the filter will be ineffective, i.e., $\frac{\pi}{4} \le T(2\pi \times 10^4)$. So, $T \ge 12.5\mu$sec.

(c)

$$\Omega = \frac{\omega}{T} \Rightarrow \Omega_c = \frac{\pi}{4T}$$



4.27. Parseval's Theorem:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

When we upsample, the added samples are zeros, so the upsampled signal $x_u[n]$ has the same energy as the original $x[n]$:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x_u[n]|^2,$$

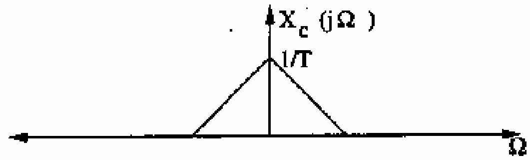
and by Parseval's theorem:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X_u(e^{j\omega})|^2 d\omega.$$

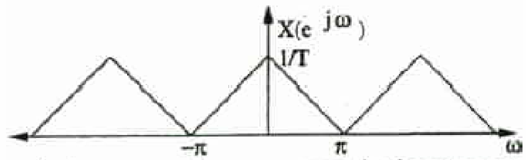
Hence the amplitude of the Fourier transform does not change.

When we downsample, the downsampled signal $x_d[n]$ has less energy than the original $x[n]$ because some samples are discarded. Hence the amplitude of the Fourier transform will change after downsampling.

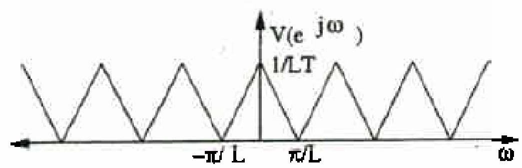
4.38. $X_c(j\Omega)$ is drawn below.



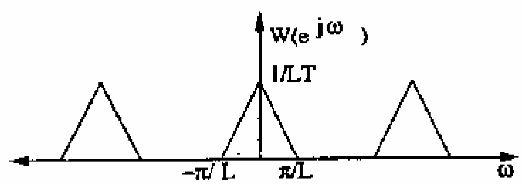
$x_c(t)$ is sampled at sampling period T, so there is no aliasing in $x[n]$.



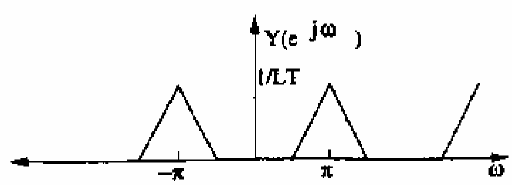
Inserting $L-1$ zeros between samples compresses the frequency axis.



The filter $H(e^{j\omega})$ removes frequency components between π/L and π.

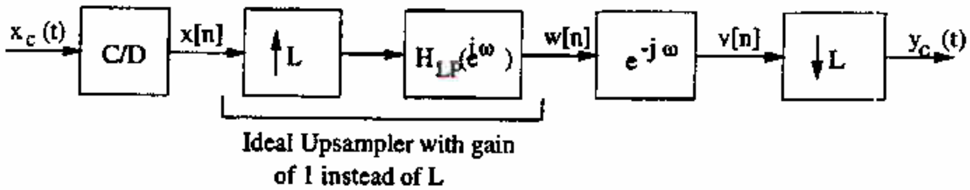


The multiplication by $(-1)^n$ shifts the center of the frequency band from 0 to π.



4.40. Split $H(e^{j\omega})$ into a lowpass and a delay.

$$
\begin{aligned}
H(e^{j\omega}) &= H_{LP}(e^{j\omega})e^{-j\omega} \\
H_{LP}(e^{j\omega}) &= \begin{cases} 1, & |\omega| < \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega| \le \pi \end{cases}
\end{aligned}
$$



Then we analyze the system as follows:

$$
\begin{aligned}
x[n] &= x_c(nT) \quad \text{no aliasing assumed} \\
w[n] &= \frac{1}{L}x_c(n\frac{T}{L}) \quad \text{rate change} \\
v[n] &= w[n-1] = \frac{1}{L}x_c\left(n\frac{T}{L} - \frac{T}{L}\right), \quad \text{delay at higher rate} \\
y[n] &= v[nL] = \frac{1}{L}x_c\left(nT - \frac{T}{L}\right)
\end{aligned}
$$