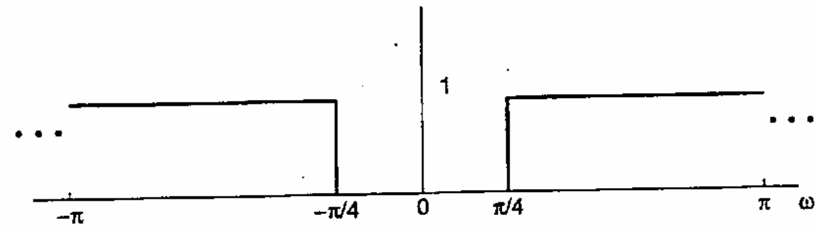
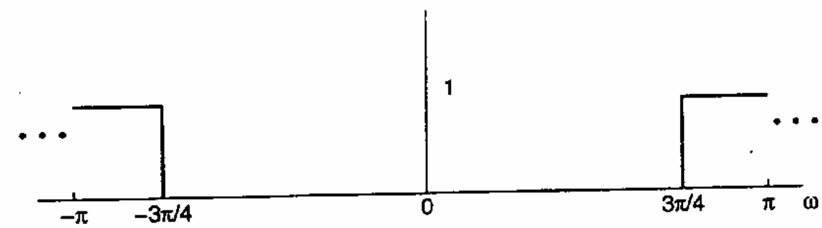
5.21. $h_{lp}[n]$ is an ideal lowpass filter with $\omega_c = \frac{\pi}{4}$

(a) $y[n] = x[n] - x[n] * h_{lp}[n] \Rightarrow H(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$
This is a highpass filter.

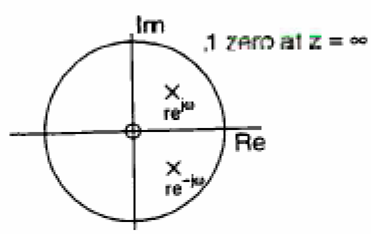


(b) $x[n]$ is first modulated by π, lowpass filtered, and demodulated by π. Therefore, $H_{lp}(e^{j\omega})$ filters the high frequency components of $X(e^{j\omega})$.
This is a highpass filter.



(c) $h_{lp}[2n]$ is a downsampled version of the filter. Therefore, the frequency response will be "spread out" by a factor of two, with a gain of $\frac{1}{2}$.
This is a lowpass filter.

5.26. (a) A labeled pole-zero diagram appears below.



The table of common z-transform pairs gives us

$$(r^n \sin\omega_0 n)u[n] \longleftrightarrow \frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}, \quad |z| > r$$

which enables us to derive $h[n]$.

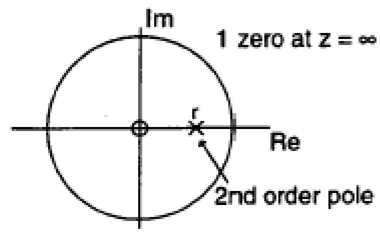
$$h[n] = \left(\frac{1}{\sin\omega_0}\right)(r^n \sin\omega_0 n)u[n]$$

(b) When $\omega_0 = 0$

$$H(z) = \frac{rz^{-1}}{1 - (2r\cos\omega_0)z^{-1} + r^2 z^{-2}} = \frac{rz^{-1}}{(1 - rz^{-1})^2}, \quad |z| > r$$

Again, using a table lookup gives us

$$h[n] = nr^n u[n]$$



5.28. (a)

$$H(z) = \frac{A}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}, \quad |z| > \frac{1}{2} \;\; h[n] \text{ causal}$$

$$H(1) = 6 \Rightarrow A = 4$$

(b)

$$\begin{aligned} H(z) &= \frac{4}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}, \quad |z| > \frac{1}{2} \\ &= \frac{(\frac{12}{5})}{1-\frac{1}{2}z^{-1}} + \frac{(\frac{8}{5})}{1+\frac{1}{3}z^{-1}} \\ h[n] &= \frac{12}{15}\left(\frac{1}{2}\right)^n u[n] + \frac{8}{5}\left(-\frac{1}{3}\right)^n u[n] \end{aligned}$$

(c) (i)

$$x[n] = u[n] - \frac{1}{2}u[n-1] \Leftrightarrow X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}}, \quad |z| > 1$$

$$\begin{aligned} Y(z) &= X(z)H(z) \\ &= \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}} \cdot \frac{4}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}, \quad |z| > 1 \\ &= \frac{4}{(1-z^{-1})(1+\frac{1}{3}z^{+1})} \\ &= \frac{3}{1-z^{-1}} + \frac{1}{1+\frac{1}{3}z^{-1}} \\ y[n] &= 3u[n] + \left(-\frac{1}{3}\right)^n u[n] \end{aligned}$$

(ii)

$$x(t) = 50 + 10\cos(20\pi t) + 30\cos(40\pi t)$$

$$T = \frac{1}{40} \quad t = nT$$

$$\begin{aligned} x[n] &= 50 + 10\cos\frac{\pi}{2}n + 30\cos\pi n \\ &= 50 + 5e^{j(n\pi/2)} + 5e^{-j(n\pi/2)} + 15e^{jn\pi} + 15e^{-jn\pi} \end{aligned}$$

Using the **eigenfunction property**:

$$y[n] = 50H(e^{j0}) + 5e^{j(n\pi/2)}H(e^{j(\pi/2)}) + 5e^{-j(n\pi/2)}H(e^{-j(\pi/2)}) + 15e^{jn\pi}H(e^{j\pi}) + 15e^{-jn\pi}H(e^{-j\pi})$$

$$H(e^{j\omega}) = \frac{4}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-j2\omega}}$$

$H(e^{j0}) = 6$, $H(e^{j(\pi/2)}) = 7\left(\frac{12}{25}\right) - j\frac{12}{25}$, $H(e^{-j(\pi/2)}) = 7\left(\frac{12}{25}\right) + j\frac{12}{25}$,
$H(e^{j\pi}) = 4$, $H(e^{-j\pi}) = 4$

$$y[n] = 300 + 24\sqrt{2}\cos\left(\frac{\pi}{2}n - \tan^{-1}\left(\frac{1}{7}\right)\right) + 120\cos\pi n$$

5.29.

$$\begin{aligned} H(z) &= \frac{21}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})(1-4z^{-1})} \\ &= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{28}{1-2z^{-1}} + \frac{48}{1-4z^{-1}} \end{aligned}$$

Since we know the sequence is not stable, the ROC must not include $|z| = 1$, and since it is two-sided, the ROC must be a ring. This leaves only one possible choice: the ROC is $2 < |z| < 4$.

(a)

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 28(2)^n u[n] - 48(4)^n u[-n-1]$$

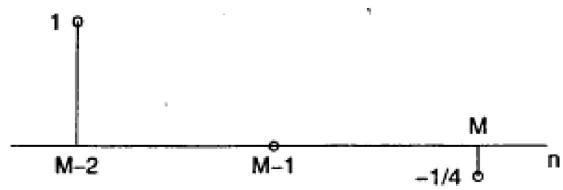
(b)

$$H_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{28}{1-2z^{-1}}$$

$$H_2(z) = \frac{48}{1-4z^{-1}}$$

5.30. (a)

$$H(z) = \frac{(z+\frac{1}{2})(z-\frac{1}{2})}{z^M} = z^{-(M-2)}\left(1-\frac{1}{4}z^{-2}\right)$$



(b)

$$\begin{aligned} w[n] &= x[n-(M-2)] - \frac{1}{4}x[n-M] \\ y[n] &= w[2n] = x[2n-(M-2)] - \frac{1}{4}x[2n-M] \end{aligned}$$

Let $v[n] = x[2n]$,

$$y[n] = v[n-(M-2)/2] - \frac{1}{4}v[n-(M/2)]$$

Therefore,

$$g[n] = \delta[n-(M-2)/2] - \frac{1}{4}\delta[n-(M/2)], \quad M \text{ even}$$

$$G(z) = z^{-(M-2)/2} - \frac{1}{4}z^{-M/2}$$