Problem 1: (30%)

A causal LTI system has system function

$$H(z) = \frac{1.28z^{-2}}{1+0.64z^{-2}} = \overset{2}{\cancel{A}} + \frac{\overset{-1}{\cancel{B}}}{1-0.8e^{j\pi/2}z^{-1}} + \frac{-1}{1-0.8e^{-j\pi/2}z^{-1}}.$$

(a) Determine A and B in the partial fraction expansion of $H(z)$ and from it determine the impulse response $h[n]$. Simplify your answer as much as possible to receive full credit. **You must determine the partial fraction expansion. Do not write down the answer from a table or other information that you may have on your note sheet.**

$$A = \frac{1.28}{.64} = \underline{\underline{2}} \qquad B = \frac{1.28 \cdot \frac{1}{(.8e^{j\pi/2})^2}}{1 - \frac{.8e^{-j\pi/2}}{.8e^{j\pi/2}}} = \frac{\frac{1.28}{.64}e^{-j\pi}}{1-e^{-j\pi}} = \underline{\underline{-1}}$$

$$h[n] = 2\delta[n] - (.8e^{j\pi/2})^n u[n] - (.8e^{-j\pi/2})^n u[n]$$

$$= 2\delta[n] - (.8)^n e^{j\frac{\pi}{2}n} u[n] - (.8)^n e^{-j\frac{\pi}{2}n} u[n]$$

$$= 2\delta[n] - 2(.8)^n \cos(\tfrac{\pi}{2}n) u[n]$$

(b) Find the input $x[n]$ so that the corresponding output of the above system is $y[n] = \delta[n]$.

$$Y(z) = 1 = H(z)X(z) \Rightarrow X(z) = \frac{1}{H(z)} = \frac{1+.64z^{-2}}{1.28z^{-2}}$$

$$X(z) = \frac{1}{1.28}z^2 + \frac{.64}{1.28} = \frac{1}{1.28}z^2 + \frac{1}{2} = .78125z^2 + .5$$

$$x[n] = .78125\,\delta[n+2] + .5\,\delta[n]$$

(c) Is the system stable? (circle correct answer) yes no. Explain.

Causal system with all poles inside the unit circle.

(d) Determine the output when the input is $x[n] = e^{j(\pi/4)n}$ for $-\infty < n < \infty$. You may leave your answer in any convenient form.

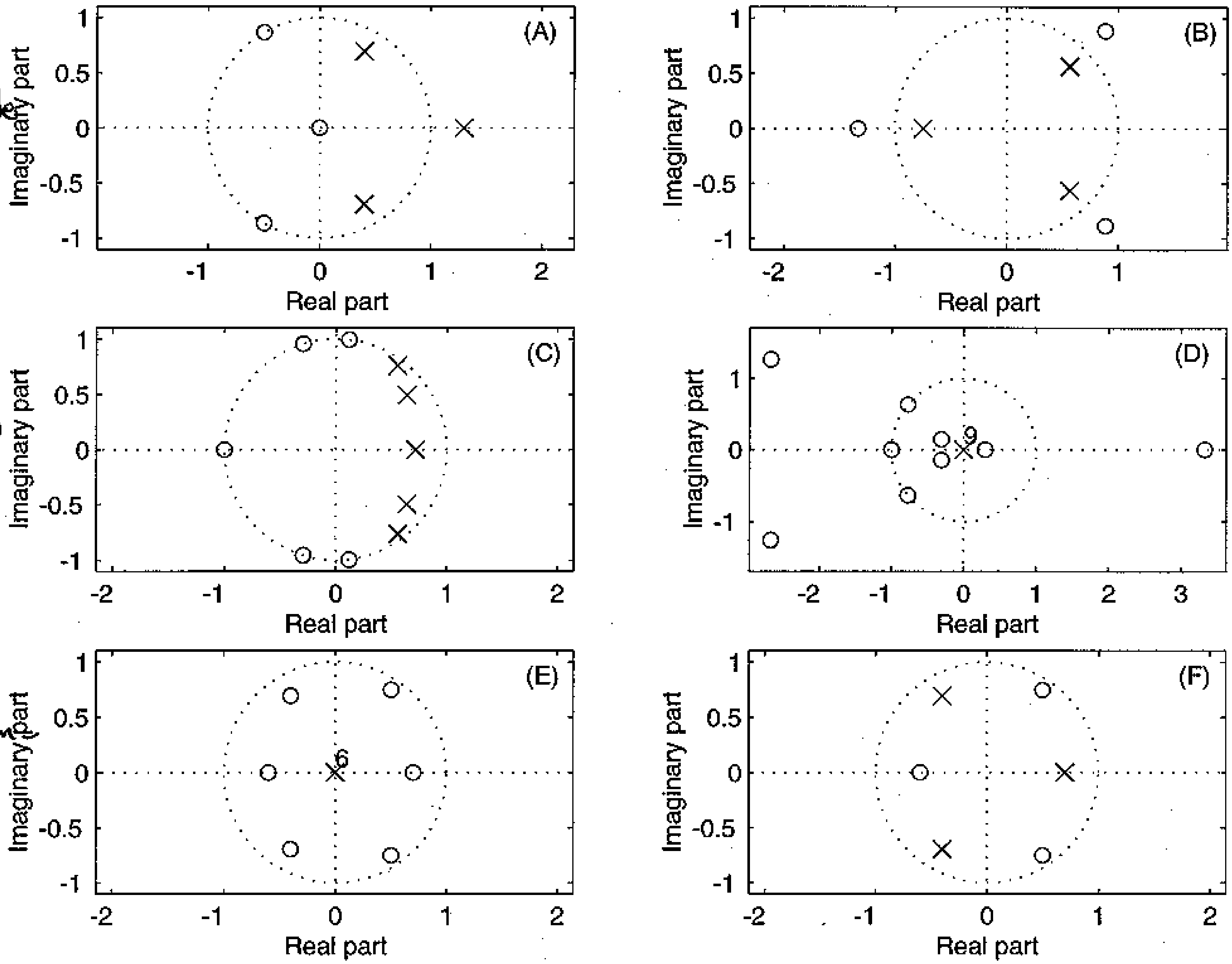
$$y[n] = H(e^{j\pi/4})e^{j\frac{\pi}{4}n}$$

$$= \frac{1.28e^{-j\pi/2}}{1+.64e^{-j\pi/2}}e^{j\frac{\pi}{4}n} = \frac{-j1.28}{1-j(.64)}e^{j\frac{\pi}{4}n}$$

$$= 1.0781\,e^{j(\frac{\pi}{4}n - 1.0015)} \qquad -\infty < n < \infty$$

Problem 4: (30%)

The following pole-zero plots describe six different causal LTI systems. **You may tear off this sheet to make it easy use it to answer the questions in Problem 4. You do not need to hand in this sheet.**



IIR unstable

IIR allpass

IIR lowpass non-min-phase

FIR linear phase lowpass

FIR minimum phase

IIR minimum phase

Problem 4: (30%)

Answer the following questions about the pole-zero plots on page 6.

(a) Which systems are IIR systems?

(A) (B) (C) (D) (E) (F) none all

Must have poles other than at $z=0$.

(b) Which systems are FIR systems?

(A) (B) (C) (D) (E) (F) none all

All poles must be at $z=0$.

(c) Which systems are stable systems?

(A) (B) (C) (D) (E) (F) none all

All poles must be inside unit circle.

(d) Which systems are minimum phase systems?

(A) (B) (C) (D) (E) (F) none all

must have all poles and zeros inside unit circle.

(e) Which systems are linear phase systems?

(A) (B) (C) (D) (E) (F) none all

All poles at $z=0$ and zeros in reciprocal pairs

(f) Which systems have $|H(e^{j\omega})|$=constant for all ω?

(A) (B) (C) (D) (E) (F) none all

Poles and zeros are in reciprocal pairs

(g) Which systems have corresponding stable and causal inverse systems?

(A) (B) (C) (D) (E) (F) none all

Requires all zeros inside unit circle

(h) Which system has the shortest (least number of non-zero samples) impulse response?

(A) (B) (C) (D) (E) (F) none all

Length is 6 for (E). (D) has length 9 and all the rest are ∞.

(i) Which systems have *lowpass* frequency responses?

(A) (B) (C) (D) (E) (F) none all

Poles build up and zeros on circle creates stopband. (A) looks to be a lowpass, but it is unstable.

(j) Which systems have minumum group delay?

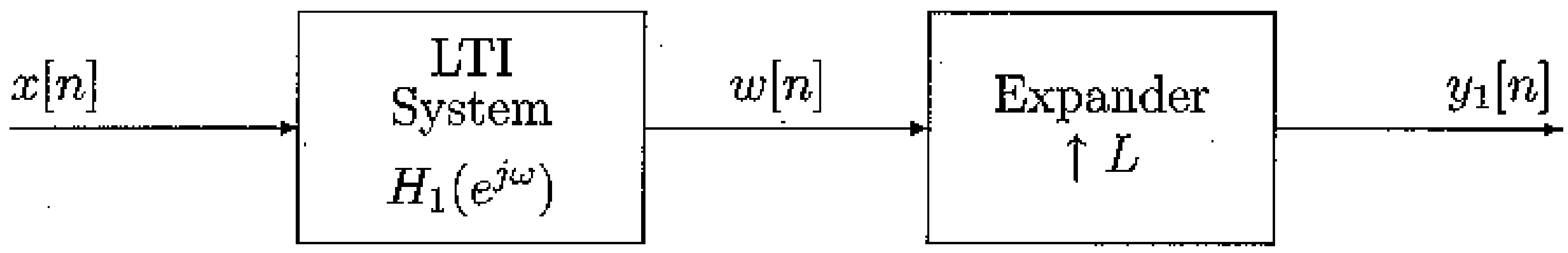
(A) (B) (C) (D) (E) (F) none all

Same as minimum phase. (In fact (d), (g) and (j) are all equivalent conditions.)

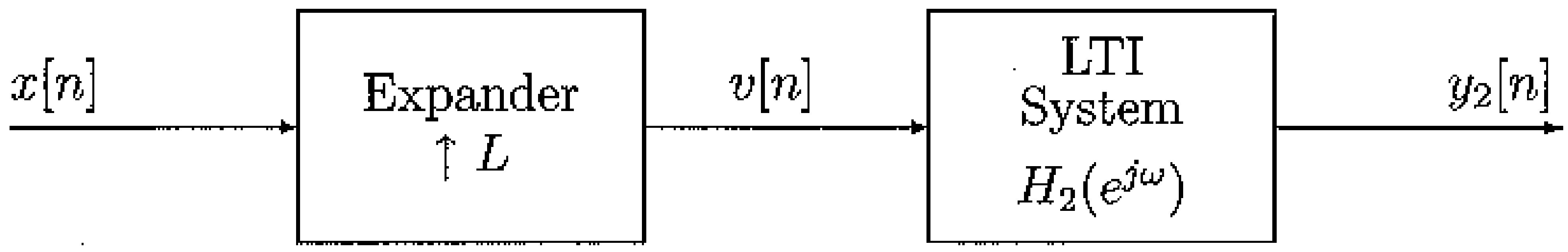
NOTE: WORK ONE OF THE FOLLOWING TWO PROBLEMS (3a or 3b). Circle the number of the problem that you wish to have me grade for the exam. If you have time, you may submit the other problem for 5% extra credit.

Problem 3a: (10%)

Consider the following cascade of an LTI discrete-time system with an expander:



Show that if the LTI system in the following diagram is chosen properly, then the two systems are equivalent; i.e., $y_2[n] = y_1[n]$ for all inputs $x[n]$.



In other words, find $H_2(e^{j\omega})$ so that $y_2[n] = y_1[n]$ for any input $x[n]$.

$$Y_1(e^{j\omega}) = W(e^{j\omega L}) = H_1(e^{j\omega L})X(e^{j\omega L})$$

$$Y_2(e^{j\omega}) = H_2(e^{j\omega})V(e^{j\omega}) = H_2(e^{j\omega})X(e^{j\omega L})$$

$$\Rightarrow H_2(e^{j\omega}) = H_1(e^{j\omega L})$$

Problem 3b: (10%)

An LTI system has system function

$$H(z) = \frac{1 + 3.5z^{-1} - 2z^{-2}}{1 + 0.64z^{-2}} = \frac{(1 - .5z^{-1})(1 + 4z^{-1})}{(1 - 0.8e^{j\pi/2}z^{-1})(1 - 0.8e^{-j\pi/2}z^{-1})}.$$

Determine the system function $H_{\min}(z)$ of a minimum phase system such that $|H(e^{j\omega})| = |H_{\min}(e^{j\omega})|$.

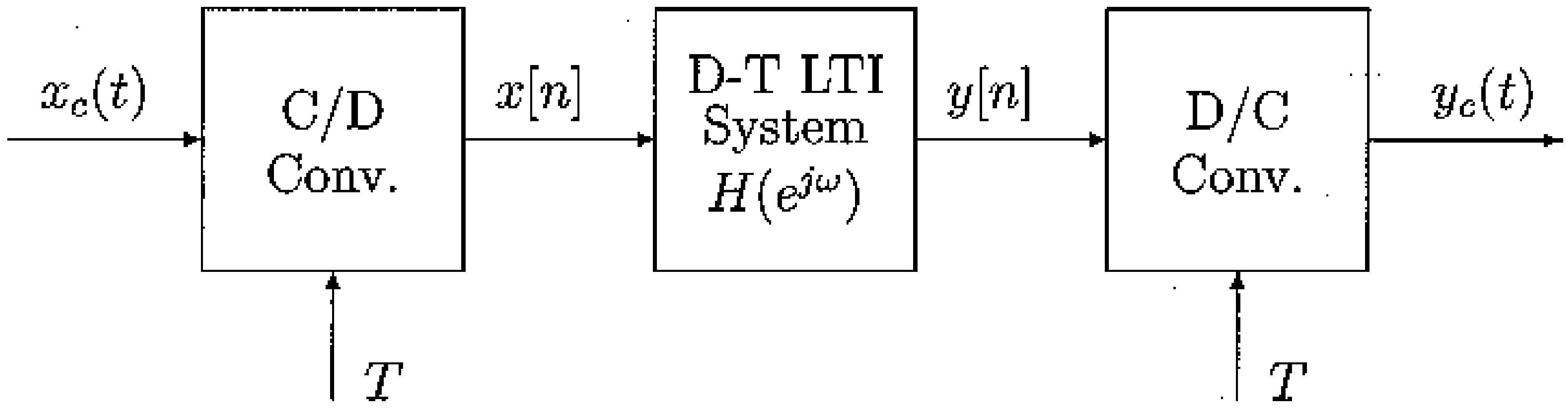
$$H(z) = H_{min}(z)\,H_{ap}(z) = \frac{(1 - .5z^{-1})(1 + \frac{1}{4}z^{-1})}{(1 - .8e^{j\pi/2}z^{-1})(1 - .8e^{-j\pi/2}z^{-1})} \cdot \frac{1 + 4z^{-1}}{1 + \frac{1}{4}z^{-1}}$$

$$H_{min}(z) = \frac{4(1 - .5z^{-1})(1 + \frac{1}{4}z^{-1})}{(1 - .8e^{j\pi/2}z^{-1})(1 - .8e^{-j\pi/2}z^{-1})} \Rightarrow |H_{min}(e^{j\omega})| = |H(e^{j\omega})|$$

$$H_{ap}(z) = \frac{z^{-1} + 1/4}{1 + \frac{1}{4}z^{-1}} \Rightarrow |H_{ap}(e^{j\omega})| = 1$$

Problem 2: (30%)

Consider the following system for discrete-time processing of the continuous-time signal $x_c(t)$.



Assume that the input signal has a bandlimited continuous-time Fourier transform such that $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(1000)$.

(a) Determine an appropriate sampling period T and a linear time-invariant discrete-time system $H(e^{j\omega})$ so that the continuous-time Fourier transform of the output satisfies the condition

$$Y_c(j\Omega) = \begin{cases} X_c(j\Omega) & |\Omega| < 2\pi(400) \\ 0 & |\Omega| \geq 2\pi(400). \end{cases} = H_{eff}(j\Omega)\, X_c(j\Omega)$$

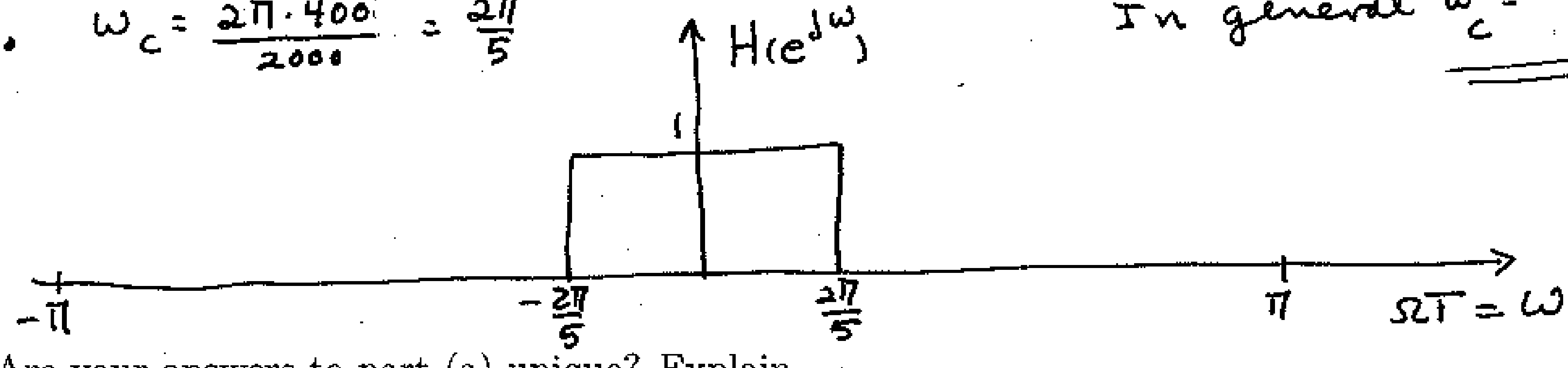
Give your answer for the LTI system as a carefully labeled plot of $H(e^{j\omega})$ as a function of ω for $-\pi < \omega < \pi$. **You must give a numerical answer for your choice of T.**

$$\Rightarrow H_{eff}(j\Omega) = \begin{cases} 1 & |\Omega| < 2\pi(400) \\ 0 & |\Omega| \geq 2\pi(400) \end{cases} = H(e^{j\Omega T})$$

We can pick T to avoid aliasing $\Rightarrow \frac{2\pi}{T} \geq 2\pi(1000)\cdot 2$

or $T = \frac{1}{2000}$ just avoids it. Anything smaller also works.

$\therefore\ \omega_c = \frac{2\pi \cdot 400}{2000} = \frac{2\pi}{5}$ In general $\omega_c = 800\pi T$



(b) Are your answers to part (a) unique? Explain.

Not unique since we can use any T such that no aliasing components are in the band $|\Omega| < 2\pi(400)$

so $\frac{2\pi}{T} - 2\pi(1000) > 2\pi(400)$ or $\frac{2\pi}{T} > 2\pi(1400)$ or $T < \frac{1}{1400}$

whatever T is used we must pick $\omega_c = 2\pi(400)T$