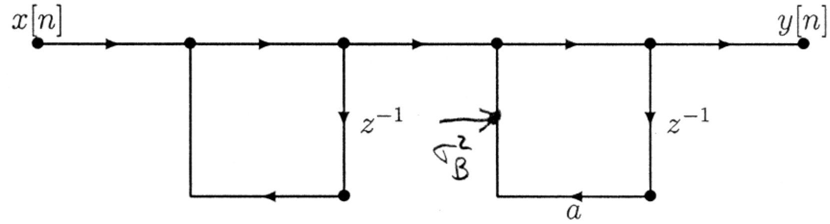
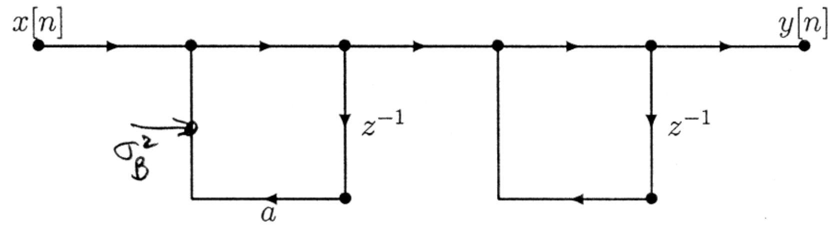
Problem 1 (30 %)

Assume that a in the following flow graphs is a real number and $0 < a < 1$. Note that under infinite-precision arithmetic, the two systems are equivalent.



Flow Graph #1



Flow Graph #2

(a) Assume that the two systems are implemented with two's-complement fixed-point arithmetic, and that in both cases all products (multiplications by 1 do not introduce noise) are immediately rounded (*before* any additions are done). Insert roundoff noise sources at appropriate locations in both flow graphs to model the rounding error. Assume that each of the noise sources inserted has average power equal to σ_B^2.

(b) If the products are rounded as described in part (a), the outputs of the two systems will differ; i.e., the output of the first system will be $y_1[n] = y[n] + f_1[n]$ and the output of the second system will be $y_2[n] = y[n] + f_2[n]$, where $y[n]$ is the output due to $x[n]$ acting alone, and $f_1[n]$ and $f_2[n]$ are the outputs due to the noise sources. Determine the power density spectrum of the output noise $\Phi_{f_1 f_1}(e^{j\omega})$. Also determine the total noise power of the output of flow graph #1; i.e., determine $\sigma_{f_1}^2$.

$$\Phi_{f_1 f_2}(e^{j\omega}) = \sigma_B^2 \cdot \left| \frac{1}{1 - a e^{-j\omega}} \right|^2 = \frac{\sigma_B^2}{1 + a^2 - 2a\cos\omega}$$

$$\sigma_{f_1}^2 = \sigma_B^2 \cdot \sum_{n=0}^{\infty} a^{2n} = \frac{\sigma_B^2}{1 - a^2}$$

(c) Without actually computing the output noise power for flow graph #2, you should be able to determine which system has the largest total noise power at the output? *Give a brief explanation of your answer.*

In the second system the noise of the first stage is amplified by the pole at $z=1$ so actually the noise power of the output is theoretically infinite.

Problem 2: (30%)

A Kaiser window $w_K[n]$ of length $M+1 = 37$ and $\beta = 5.653$ is used to design a linear-phase lowpass filter with frequency response $H_{\rm lp}(e^{j\omega})$. This filter meets the following specifications:

$$.999 \quad 0.99 \le |H_{\rm lp}(e^{j\omega})| \le 1.001, \qquad 0 \le |\omega| \le 0.2\pi \qquad \Delta\omega = .2\pi$$
$$|H_{\rm lp}(e^{j\omega})| \le 0.001 \qquad 0.4\pi \le |\omega| \le \pi. \qquad \delta = .001$$

Now a bandpass filter is designed with the *same* Kaiser window by applying it to the impulse response $h_d[n]$ whose ideal frequency response is

$$H_d(e^{j\omega}) = \begin{cases} 0 & |\omega| < 0.2\pi \\ 10e^{-j\omega n_d} & 0.2\pi < |\omega| < 0.6\pi \\ 0 & 0.6\pi < |\omega| \le \pi. \end{cases} \tag{1}$$

That is, a linear-phase FIR highpass filter with impulse response $h[n] = w_K[n]h_d[n]$ and frequency response $H(e^{j\omega})$ was obtained by multiplying $h_d[n]$ by the same Kaiser window $w_K[n]$ that was used to design the first mentioned lowpass filter.

(a) How should the delay n_d of the ideal filter be chosen so that the resulting FIR filter has linear phase?

$$n_d = \frac{M}{2} = 18$$

(b) Determine the impulse response $h_d[n]$ of the ideal bandpass filter in (1) above for the value of n_d found in part (a). *Note that it is not necessary to work this part in order to work parts (c) and (d).* $H_d(e^{j\omega})$ is the difference between two lowpass filters. i.e.

$$h_d[n] = 10\frac{\sin .6\pi(n-18)}{\pi(n-18)} - 10\frac{\sin .2\pi(n-18)}{\pi(n-18)}$$

(c) The resulting FIR bandpass filter meets a set of specifications of the following form:

$$\begin{aligned} |H(e^{j\omega})| &\le \delta_1 & 0 \le |\omega| \le \omega_1 \\ G - \delta_2 \le |H(e^{j\omega})| &\le G + \delta_2 & \omega_2 \le |\omega| \le \omega_3 \\ |H(e^{j\omega})| &\le \delta_3 & \omega_4 \le |\omega| \le \pi \end{aligned}$$

Use information from the lowpass filter specifications to determine the values of ω_1, ω_2, ω_3, ω_4, δ_1, δ_2, δ_3 and G. $\delta_1 = \delta_2 = \delta_3 = 10\cdot(.001) = .01$

$\omega_1 = .2\pi - \frac{\Delta\omega}{2} = .1\pi \quad \omega_2 = .2\pi + \frac{\Delta\omega}{2} = .3\pi \quad \omega_3 = .6\pi - \frac{\Delta\omega}{2} = .5\pi$

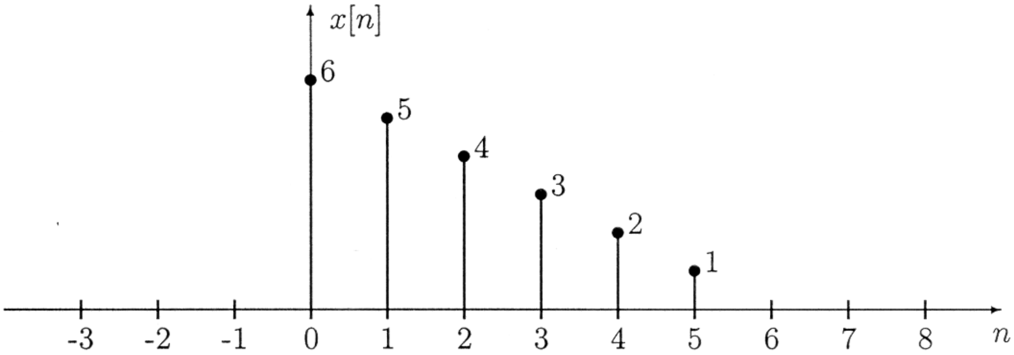
$\omega_4 = .6\pi + \Delta\omega/2 = .7\pi$ and $G = 10$ Note that you can get all you need from H_{lp}

(d) If we increase the window length to $M_1 + 1 = 73$ keeping β the same, which parameters found in (c) will change significantly? Give the new values of those parameters.

Doubling M will halve $\Delta\omega$ so

$\omega_1 = .15\pi$, $\omega_2 = .25\pi$, $\omega_3 = .55\pi$, $\omega_4 = .65\pi$

Problem 3: (40%)
Consider the 6-point sequence $x[n] = 6\delta[n]+5\delta[n-1]+4\delta[n-2]+3\delta[n-3]+2\delta[n-4]+\delta[n-5]$.

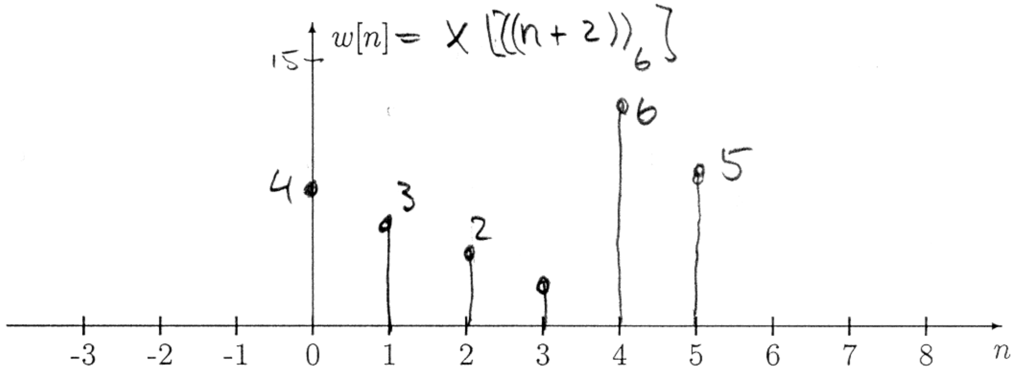


(a) Determine $X[k]$, the 6-point DFT of $x[n]$. Express your answer in terms of $W_6 = e^{-j2\pi/6}$.

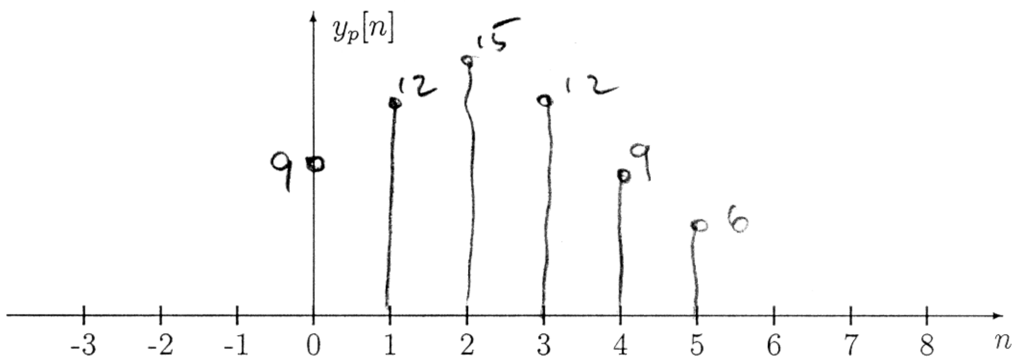
$$X[k] = 6 + 5W_6 + 4W_6^2 + 3W_6^3 + 2W_6^4 + W_6^5$$

(b) On the axes below, plot the sequence $w[n]$, $n = 0, 1, \ldots, 5$, that is obtained by computing the inverse 6-point DFT of $W[k] = W_6^{-2k}X[k]$.

$w[n] = x[((n+2))_6]$



(c) Now use any convenient method to evaluate the **6-point circular** convolution of $x[n]$ with the sequence $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$. Call your answer $y_p[n]$ and plot it below to receive full credit.



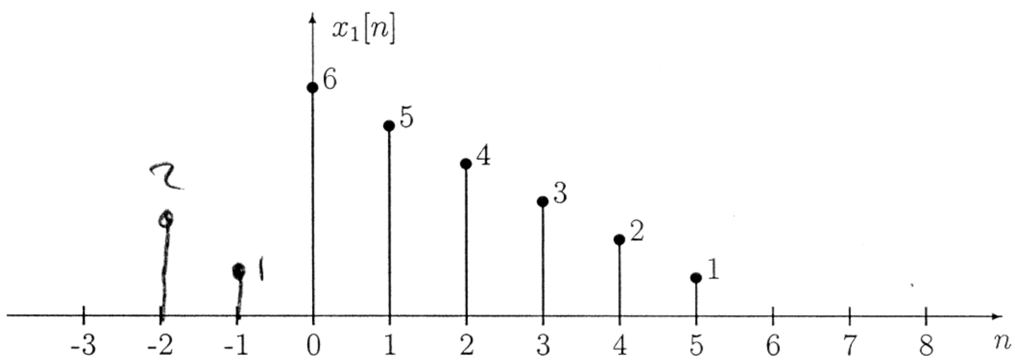
(d) If we convolve the given $x[n]$ with the given $h[n]$ by N-point circular convolution, how should N be chosen so that the result of the circular convolution is identical to the result of ordinary (not circular) convolution? That is, choose N so that

$$y_p[n] = \sum_{m=0}^{N-1} x[m]h[((n-m))_N] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \quad \text{for } 0 \leq n \leq N-1.$$

The regular convolution will have $6+3-1=8$ points Therefor $N \geq 8$ gives the desired result

(e) In certain applications, the *ordinary* convolution of a finite-length signal $x[n]$ of length L samples with a shorter finite-length impulse response $h[n]$ is required to be identical (over $0 \leq n \leq L-1$) to what would have been obtained by L-point circular convolution of $x[n]$ with $h[n]$. This can be achieved by augmenting the sequence $x[n]$ appropriately. On the graph below where $L = 6$, add samples to the given sequence $x[n]$ to produce a new sequence $x_1[n]$ such that with the given sequence $h[n]$, the ordinary convolution $y_1[n] = x[n] * h[n]$ satisfies the equation

$$y_1[n] = x[n]*h[n] = \sum_{m=-\infty}^{\infty} x_1[m]h[n-m] = y_p[n] = \sum_{m=0}^{5} x[m]h[((n-m))_6] \quad \text{for } 0 \leq n \leq 5.$$



This augmented input will give the same result as 6 conv, over $0 \leq n \leq 5$